

## Effect of partial edge load on simply supported FRP composite panel for various ply orientations through dynamic approach

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### Abstract

Laminated composite plates do have very high strength to weight ratio compared to conventional metal components but there is a disadvantage due to its slenderness. Slenderness poses a problem of instability. A rigorous study of stability of such components needs to be done before designing those components for various loading and boundary conditions. If the load acting on the component is independent of configuration of component, it results in static instability. In this case, as the load on the structure increases the natural frequency of structure decreases and at one particular load it reduces to zero. This load is known as static buckling load.

In this paper, a framework to study the effect of partial edge load on various ply orientations on stability of simply supported FRP composite panel through dynamic approach is presented. MatLab code is developed based on classical Lamination theory and Finite element approach. This kind of work requires the validation of formulation and coding using the principle of part to whole. Validation of formulation and implementation was conducted by various load and boundary conditions. Finally the parametric study is done and results have been found interesting which are reported here.

**Keywords:** Partial edge load, buckling, finite element, laminated composite panels.

### 1 Introduction

In the past decade composite materials have become critical elements for the development of vehicles of transportation, machines, marine structures and space structure. The utilization of overlaid composites in outlining these structural elements has brought about a huge improvement in weight reduction, payload, mobility and durability. A laminated composite is a heap of strata of fiber-strengthened laminates. Matrix and fibers are combined together to form the fiber reinforced laminates.

With the expanding utilization of composite materials, the requirement for the advanced techniques for examination and testing have become critical. In

composite materials, transverse stresses and strains strongly impact their behavior. Specifically, the transverse shear stress impacts are more proclaimed. The Classical Laminate plate hypothesis, which is not planned to represent the impact of these stresses, is not applicable to orthotropic plate investigation.

The basic components of any structures like plates, shells and bars are once in a while subjected to occasional in-plane load and turn out to be dynamically unstable for specific combinations of frequencies and load amplitude. Such a phenomenon is known as parametric resonance or dynamic instability.

Composite laminate failure could be traced in an assortment of methods. When a layer in a composite fails in a direction parallel to that of fiber or perpendicular to the fiber orientation it is the First Ply Failure (FPF). Last Ply Failure (LPF) happens when the material no longer has the capacity to carry any additional loads.

### FRP Composites

Fibre Reinforced Plastic (FRP) is the common term for a exclusively adaptable group of composites that are present everywhere from chemical plant to amenity power boats. The FRP structure contains unsaturated polyester (UP) resin moulded in blend with reinforcing material. Major reason for the strength of FRP Composites is the type, quantity and organization of the reinforcing fibre; more than 90% of the reinforcing materials used are glass fibers. The major benefit is Light weight with high strength-to-weight ratio. The polymer resin is characteristically viscous, and may be moulded easily with high range of stiffness, better chemical resilience, better electrical insulation properties and dimensional stability in a wide range of temperatures.

The growing application of slender elements in aircraft, civil, marine and offshore structures necessitates precise and effectual examination of their stability under various loads. Modern development of Carbon Fiber composites, with their lower mass and stiffness, has also enhanced to the progress of light systems. New generation of aircrafts will be employing superior composites in their

key structural systems.

The static equilibrium state of an actual structure or its element is a stable one, to which the arrangement reverts (returns) after being marginally disturbed. Occasionally, though, the equilibrium position is the one which is unstable, from which the arrangement goes to buckle after being faintly disturbed [1].

## 2 Methodology

### 2.1 Stability of Static Equilibrium

It is assumed that the structure under consideration is elastic, since some effects such as plasticity or creep introduce difficulties like path requirement, which are beyond our scope. The applied forces are categorised by a load factor  $\lambda$ , also termed as a load parameter. This magnitude scales reference loads to deliver the tangible applied loads. Fixing  $\lambda = 0$  means that the structure is unloaded and has taken equilibrium pattern  $C(0)$  termed as the undeformed state. It is assumed that this state is stable. As  $\lambda$  is monotonically varied the structure distorts and attains equilibrium state  $C(\lambda)$ . These are assumed to be (i) unceasingly reliant on  $\lambda$ , and (ii) stable for adequately small  $|\lambda|$ .

### 2.2 Stability of Dynamic Equilibrium

Stability is a topic that comprises the static case as a specific one. Presume that a machine-driven system is moving in a foreseeable manner. An example of dynamic equilibrium is a bridge or tower which oscillates by the influence of wind, an aircraft is flying a predefined trajectory under automatic pilot, a satellite rotates around Earth and the Earth rotates around the Sun. What is the compassion of such a motion to variation of constraints like initial state or force amplitude? If the system involves stochastic or chaotic elements, like commotion, the analysis may also need probabilistic methods.

### 2.3 Buckling

In engineering structures there are many modes of failures. Some of them include alternate stresses, buckling, bending, fatigue, creep etc. We can observe huge difference between the distortions of the plate where equilibrium configuration of the plate changes into non-flat configuration when the load is enlarged. If the loads applied to a flat plate are minor, then there is no detectable distortion. Thus, the plate becomes unstable in the case of enlarged load condition. Buckling takes place in plates, columns, shells, and other structural elements of uniform or non-uniform geometry. It is called as critical buckling load when the least load at which the equilibrium is distressed. Thin flat plates carry axial compressive loads where it is observed that fibre reinforced composites are not in the form of columns. Buckling may occur in composite plates due to a load beyond a safety value in compressive loads which may

also cause disastrous failures. To analyse the behavior of the composite under loads and solving to find the buckling load, there are many theoretical computations which formulate the behavior of such composites.

Buckling of a plate structure can induce an unacceptable degradation in the aerodynamic profile of a space vehicle. It can activate general buckling of a larger structure because of a redeployment of loads; it can also affect the response to the structure adequately to cause failure from excessive displacement or fatigue or aero elastic phenomena [2].

An inspection of the buckling performance of a single plate supported along its edges is an important preliminary step towards consideration of local buckling behaviours of plate assemblies. The buckling stresses are obtained from the concept of divergence of an initially perfect structure. In practice, the response of the structure is incessant, due to the inevitable presence of initial imperfections. Thus the critical stress is best viewed as a useful index to the behaviour, as slender plates can continue to carry additional loads well after initial buckling.

It is common to use the elastic critical buckling stress as a target for delineating dissimilar forms of plate buckling: if material yielding occurs prior to the elastic critical buckling stress, this is known as inelastic buckling; the strength at magnitudes greater than the elastic critical buckling stress, and the associated distortions that occur under such loading, are referred to as post-buckling and may be either elastic or inelastic. Finally, ultimate strength refers to the extreme load the plate may carry, typically independent of deformation, which may indeed be quite large.

Actual plate response under load is more complex than the simple motions of inelastic buckling and post-buckling; this is due, in part, to inevitable imperfections. In an imperfect plate out-of-plane distortions begin directly upon loading; such distortions lead to second order (geometrically nonlinear) forces (and strains) that must be accounted for throughout the loading/deformation; and thus the motions of buckling and post-buckling are not definitively distinct. Under load the stress field response of a thin plate is complex and varies along the length, across the width, and through the thickness of the plate. A residual stress that may exist in the plate further complicates the response. A plate with an applied stress well below the elastic critical plate buckling stress may still have portions of the plate yielding; thus determining a definitive regime where a plate enters inelastic buckling is difficult. For steady yielding metals (e.g., aluminum, stainless steel) the difference between elastic and inelastic buckling becomes even more problematic.

The formulation for buckling problem is primarily developed from Hamilton's principle. For the implementation of this rule, the required basic components are: (a) the energies developed due to linear

and non-linear strains in plate, (b) kinetic energy of the plate, (c) work done in the plate by the forces applied on plate and (d) stress field in the plate [3].

### 2.4 Classical lamination theory

The mathematical formulation of laminate is derived from the formulation of a lamina. The theory used for this formulation is Classical Lamination Theory. The theory is also known as Classical Thin Lamination Theory. Stress and deformation hypothesis are two important parts of the Theory. The governing differential equation for a buckling problem is written as

$$[K_e]\{d\} - P[K_G]\{d\} = \{0\} \tag{1}$$

Here  $[K_e]$  is elastic stiffness matrix,  $\{d\}$  is displacement vector and  $[K_G]$  is geometric stiffness matrix. The eigen values obtained from the equation will give buckling loads 'P' for various modes from which the most reduced buckling load is considered as fundamental critical load of the structure.

Composite laminate is made up of 'nl' number of layers. Stress-strain behavior is found out for a  $s^{th}$  layer at a distance 'z' units from middle layer. Later stress strain variation across the thickness of individual lamina is found out by integrating over thickness of respective lamina. At last a relation is developed between strain and curvature of a laminate to laminate moments and forces by considering the effect of all laminas. The term relating laminate forces and moments with strain-curvature of laminate is known as laminate stiffness [4].

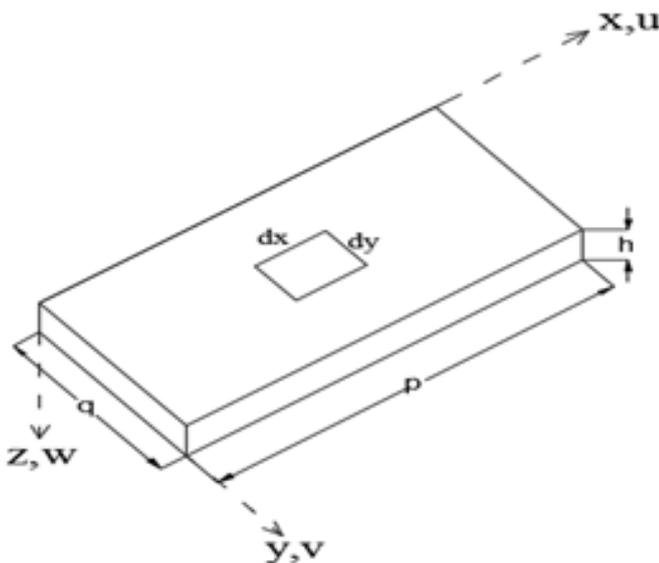


Figure 1: Layout of Composite Plate.

### 2.5 Finite element formulation

The solution for differential equations require displacement field which is admissible for the problem. Selection of admissible field is easy for simple structures,

but the same becomes complicated for complex structures. Different edge and loading conditions in case of complicated structure makes it complex to select the admissible displacement field. The problem can be effectively solved by finite element technique where the structure is divided into numerous small elements. Selection of suitable displacement field for these finite elements is very simple and easy. After developing the relation between the elements, the governing differential equations are developed to solve the problem [5].

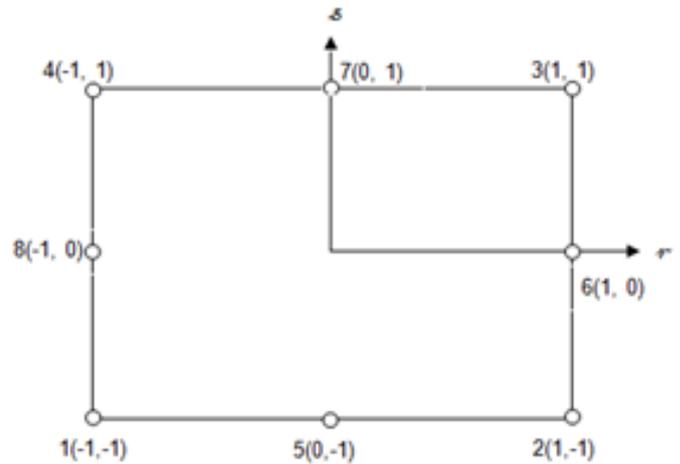


Figure 2: Eight Nodded element.

### 2.6 Eight noded rectangular isoparametric plate element:

Basically a plate is a two dimensional constituent in a structure, where one requires choosing one of the suitable two dimensional elements as shown in Figure 1. Here, discretization of the plate is done by considering an eight noded rectangular element as shown in Figure 2. Let the five degrees of freedom of each node be  $u, v, w, \theta_x, \theta_y$ . Element situated in natural co-ordinate system is shifted to Cartesian co-ordinate by utilizing jacobian matrix. In this analysis of slender plates, for derivation of element shape function we use the interpolation polynomials in which the elements are expected to possess mid-surface nodes.

## 3 Results And Discussions

The behaviour of the plate for various edge loads has been studied and is presented here. Figure 3 shows the typical composite plate with its respective co-ordinates and dimensions. Modular ratio ( $\omega$ ) =  $E_1/E_2$  is taken as 40. Aspect ratio ( $\alpha$ ) =  $p/q$  is 1.0. Poisson's ratio  $\nu_{12} = \nu_{21} = 0.25$ .  $G_{12} = G_{13} = 0.6E_2$  and  $G_{23} = 0.5E_2$ .

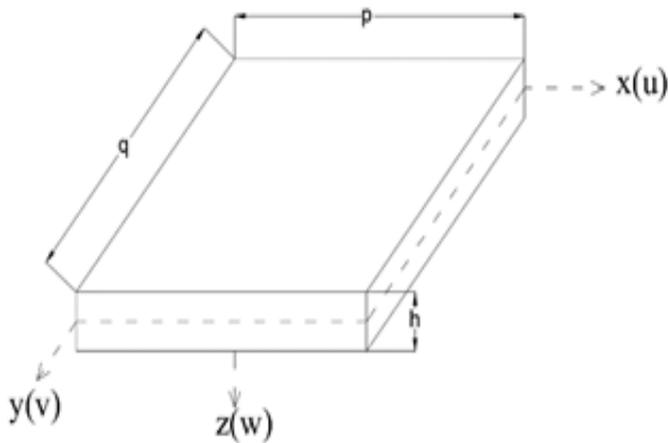


Figure 3: Typical Composite plate.

### 3.1 Variation of buckling load for various patch loads and ply orientation in a S-S-S-S laminated composite plate

Variation of buckling load is studied for various patch load values of 0.2, 0.4, 0.6 and 0.8 for all simply supported laminated composite plate and is shown in Figure 4 to Figure 7. The orientation of plies is varied from 10° to 90°. The maximum non dimensional buckling load is for anti-symmetric plate than that of symmetric plate of same number of plies, and it increases with the increase in number of plies.

#### 3.1.1 Patch Load 0.2

The variation of buckling load for a patch load of 0.2 is shown in Figure 4. Two layered anti-symmetric laminated plate shows lesser values than other layered laminates. Non dimensional buckling load is maximum at 40° ply orientation for 4 and 8 layered symmetric and anti-symmetric laminated composite plates. The lowest value of  $\gamma$  is observed for 90° ply orientation for all layer cases.

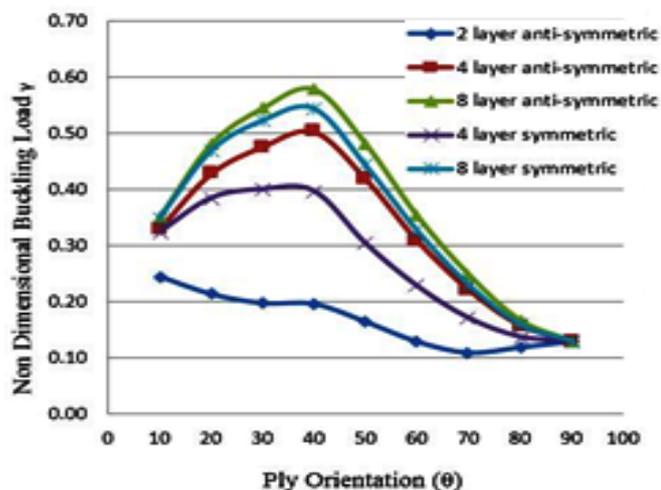


Figure 4: Variation of buckling load for different ply orientations under patch load of 0.2 (S-S-S-S Boundary condition,  $\omega = 40, \alpha=1.0$ ).

#### 3.1.2 Patch Load 0.4

The variation of buckling load for a patch load of 0.4 is shown in Figure 5. Two layered anti-symmetric laminated plate shows lesser values than other layered laminates. Maximum non dimensional buckling load is at 40° ply orientations for 2 layered anti-symmetric, 4 and 8 layered symmetric and anti-symmetric laminated composite plates. The lowest value of  $\gamma$  is observed for 90° ply orientation for all layer cases.

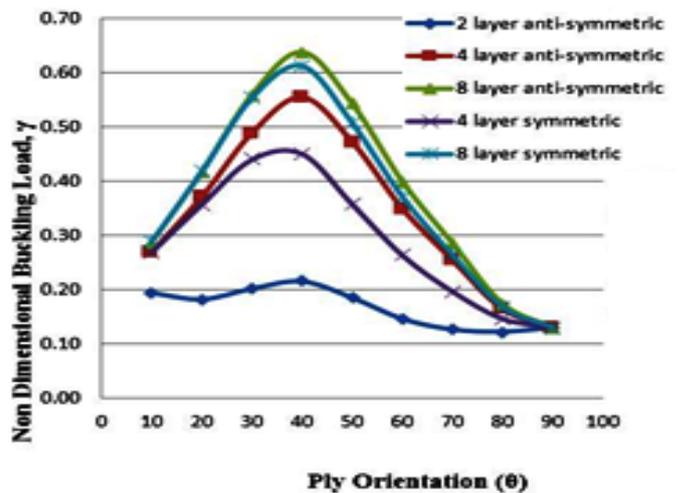


Figure 5: Variation of buckling load for different ply orientations under patch load of 0.4 (S-S-S-S Boundary condition,  $\omega = 40, \alpha=1.0$ ).

#### 3.1.3 Patch Load 0.6

The variation of buckling load for a patch load of 0.6 is shown in Figure 6. Two layered anti-symmetric laminated plate shows lesser values than other layered laminates. Maximum non dimensional buckling load is at 40° ply orientations for 2 layered anti-symmetric, 4 and 8 layered symmetric and anti-symmetric laminated composite plates. The lowest value of  $\gamma$  is observed for 90° ply orientation for all layer cases. The maximum non-dimensional buckling load is higher than that obtained for a patch load of 0.4.

#### 3.1.4 Patch Load 0.8

The variation of buckling load for a patch load of 0.8 is shown in Figure 7. As observed for previous cases two layered anti-symmetric laminated plate shows lesser values than other layered laminates. Maximum non dimensional buckling load is observed in between 40° and 50° ply orientations for 2 layered anti-symmetric, 4 and 8 layered symmetric and anti-symmetric laminated composite plates. The lowest value of  $\gamma$  is observed for 90° ply orientation in all layer cases. The maximum non dimensional buckling load is higher than that obtained for a patch load of 0.6.

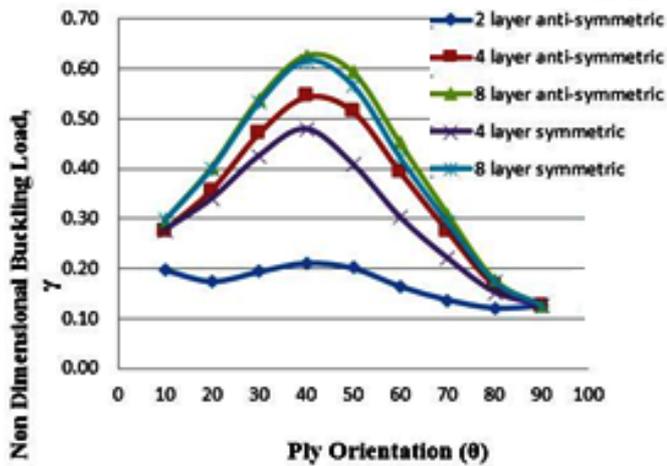


Figure 6: Variation of buckling load for different ply orientations under patch load of 0.6 (S-S-S-S Boundary condition,  $\omega = 40, \alpha=1.0$ ).

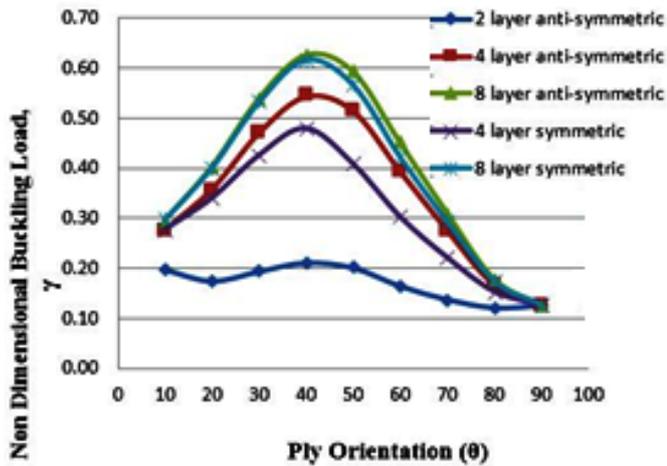


Figure 7: Variation of buckling load for different ply orientation under patch load of 0.8 (S-S-S-S Boundary condition,  $\omega = 40, \alpha=1.0$ ).

## 4 Conclusion

Composite materials have become very efficient replacement over the conventional materials in many of the structural components. Buckling analysis of these components made up of composites is very complex in comparison with the components made out of isotropic

materials. All round simply supported composite plate is considered for the investigation and effect of edge loads 0.2, 0.4, 0.6 and 0.8 on 8-layer anti-symmetric and symmetric, 4-layer anti-symmetric and symmetric and 2-layer anti-symmetric composite laminate has been investigated. A code has been developed using MatLab. From this study the following conclusions have been drawn:

Maximum buckling load is observed for 8-layered anti-symmetric composite plate for all the edge load conditions. Minimum buckling load is observed for 2-layered anti-symmetric composite plate for all the edge load conditions. Maximum buckling load is observed in between 40° and 50° Ply orientations for all edge load conditions and number of layers.

The present study gives details for S-S-S-S boundary condition and it can be extended for other boundary conditions without change in the coding. Further, the study can be carried out for composite plate with cut out by incorporating small changes in code for mesh generation. In this work the analysis has been carried out for uniformly distributed patch load and it can be extended for uniformly varying patch load.

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